PUBLIC DEBT, INFLATION AND THE COORDINATION OF FISCAL AND MONETARY POLICIES

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INTRODUCTION

In most industrial countries two distinct institutions, the central bank and the government, design and implement fiscal and monetary policies. The degree of independence between the central bank and the government varies among countries depending on historical and institutional considerations. For instance the Deutsche Bundesbank and the Federal Reserve Bank have a well established reputation of independence compared to the Bank of England and the Bank of France. But even these latter retain some autonomy in choosing their operating procedures and instruments. Thus the fiction of a single policymaker underlying the traditional approaches to the optimal coordination between fiscal and monetary policies should be abandoned.

The design of monetary and fiscal policies is less a question of coordination of alternative objectives and instruments than a question of cooperation between two authorities having their own objectives and preferences.

In both monetary and fiscal fields the financing of the budget deficit can raise two kinds of conflicts between the central bank and the Treasury (1) :

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(1) Treasury, fiscal authorities and government are used to indicate the authority which implements fiscal policy. Central bank and monetary authorities indicate the authority which implements monetary policy.

i) Both authorities recognize the merits of budget deficits in stimulating economic activity. Nevertheless the central bank in charge of reducing inflation, refuses to monetize the deficit. The Treasury in charge of the public debt management wishes to slow down the growth of public debt. If the Treasury can obtain some creation of monetary base against its liabilities it minimizes the nominal growth of the stock of public debt and thus alleviates the real burden of public debt outstanding. In this view the method used to finance the budget deficit gives rise to conflicting objectives between the central bank and the government. Until the end of the 1970's this conflicting situation prevailed in some Western countries since central banks there exhibited a strong aversion to inflation. And despite the potential crowding out effects of fiscal deficits the governments were reluctant to reduce significantly their bond issuance. Therefore the conflict was eventually resolved in favor of the central bank : the preference of both authorities led to a decreasing monetary financing of the fiscal deficit and to a correlative increase of bond financing.

ii) Both the central bank and the government wish to reduce the public debt outstanding. As far as the government is concerned the reduction of the public debt should occur through an ongoing monetization of the public deficit. From the viewpoint of the central bank only a decrease in the budget deficit can achieve both the objective of reducing inflation and that of reducing the public debt outstanding. Since the beginning of the 1980's this kind of conflict has dominated the relationship between central banks and governments, particularly in the United States. After Tabellini (1986b), our purpose here is to analyse the strategic interactions between monetary and fiscal authorities in their attempts to reduce the public debt stock.

Using a linear quadratic dynamic game this paper proposes an analysis of the influence of the strategic behavior of policymakers on the time path of public debt.

In a first part, we assume that the behavior of the private sector does not interfere in the conflict between the two authorities. The evolution of the public debt is compared under different institutional settings. We show that the cooperation between the two authorities leads to the fastest reduction of public debt outstanding. In a second part, the behavior of the private sector is explicitly modelled. We assume that the private sector anticipates inflation through an adaptive mechanism. Taking this behavior into account, both authorities have to modify their strategies as a result of the trade-off between inflation and public debt.

I. Monetary Authorities vs Fiscal Authorities

In this scenario the private sector is assumed to be neutral vis à vis the strategic interaction between monetary and fiscal authorities. The central bank and the Treasury share a common goal : the reduction of the nominal public debt outstanding. But the central bank wishes to limit money creation whereas the Treasury wants to boost the economy through a budget deficit. Thus the central bank and the Treasury disagree on the way of reducing the public debt outstanding and face the following dilemma : either they cooperate to slow down the growth of the public debt but forego their own objectives ; or they forswear cooperation in order to achieve their own objectives, in the hopes that the other policymaker will bear the burden of reducing the public debt. In doing so they may fail to reduce rapidly the growth rate of the public debt. Thus the time path of public debt depends on the monetary and fiscal objectives of the central bank and the government and on the institutional framework determining relation between the two.

Within a linear quadratic dynamic game à la Tabellini, we derive the sustainable equilibria associated with various institutional settings. We then compare the equilibria with respect to the different weights assigned to the objectives of both policymakers.

1.1. The Model

The government budget constraint gives the law of motion of the public debt :

$$(1)\quad d_{t+1} = rd_t + f_t - m_t$$

All variables are scaled to nominal income ; d denotes the public debt ; f is the budget deficit net of interest payment ; m is the monetary base issued against the liabilities of the Treasury ; r is equal to $(1+r^*)/(1+g)$ where r^* is the real rate of interest after taxes and g is the rate of growth of real income. The central bank controls the creation of monetary base whereas the budget deficit is in control of the Treasury.

The objectives of central bank and of the Treasury are described by the following quadratic loss functions :

$$(2)\quad L^M = 1/2 \left[\sum_{t=1}^{T} \alpha^t \left[m^2_t + \tau d^2_t \right] + \alpha^{T+1} \tau d^2_{T+1} \right]$$

$$(3)\quad L^F = 1/2 \left[\sum_{t=1}^{T} \beta^t \left[f^2_t + \Gamma d^2_t \right] + \beta^{T+1} \Gamma d^2_{T+1} \right]$$

According to (2) and (3) both policymakers wish to minimize deviations of the public debt from zero. This behavior is consistent with the absence of lump sum taxes. A larger public debt induces larger tax levies in order to pay interest on the public debt. In the absence of lump sum taxes taxation introduces distorting effects on the labor market. The public debt target is normalized to zero in the objective functions. Assigning two different targets d^F and d^M leads to more inertia in the evolution of the public debt without changing the results of the conflict between the two authorities.

In addition the central bank wishes to stabilize the growth of the monetary base. The chosen target normalized to zero is consistent with objectives such as the control of inflation or external balance target. The parameter τ reflects the structure of the central bank preferences and its degree of independence vis à vis the government. When τ tends to zero the central bank is definitely independent : it cares only about its monetary target. Conversely when τ tends to infinity the central bank merely finances the budget deficit chosen by the fiscal authorities. The parameter α represents a time-preference for the present factor as regards the central bank.

The fiscal authorities on the other hand wish to minimize deviations of budget deficits net of interest payments. The budget target reflects the macroeconomic stimulus desired by the government, possibly dictated by some electoral considerations. Here again the budgetary target is normalized to zero (2). The parameter Γ indicates the relative weight assigned to the debt target relative to the budget target. The parameter β represents a time-preference-for-the-present factor as regards the Treasury.

The formulation of these loss functions L^M and L^F seems rather peculiar, since the preferences of policymakers are usually expressed in terms of the ultimate goals of economic policy. Thus we make a crucial assumption : the relationship between final macroeconomic objectives and monetary and fiscal instruments is invariant through time.

1.2. Monetary and fiscal strategies without precommitment

Let us assume a game with complete information. The central bank knows the loss function of the fiscal authorities and vice versa. In this hypothesis neither of the two authorities can commit itself to a given economic policy. The two authorities simultaneously choose the sequence of their instruments $\{m_t\}_{t=0}^{t=T}$ and $\{f_t\}_{t=0}^{t=T}$. This choice of instruments in period t determines the level of the public debt in period t+1. But this level in period t+1 will influence the choice of policy instruments in period t+1. Thus when determining m_t, the central bank takes into account its influence on f_{t+1}. Conversely fiscal authorities will choose f_t knowing its impact

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(2) With target not normalized to zero the time path of public debt would be for the benchmark simulation :

$d_{t+1} = x\, d_t + y(\bar{f} - \bar{m}) - z(d^F - d^M)$ where x is the solution of the game in a closed loop Nash framework and y and z are positive constant. Therefore x is independent of the targets $\bar{f}$, $\bar{m}$, d^F and d^M.

on m_{t+1}.

The monetary strategy (resp fiscal strategy) is the optimal response to the fiscal strategy (resp monetary strategy). Thus, we get a closed loop Nash Equilibrium. In order to ease the interpretation we have reduced the game to a two period game.

From (2) and (3) we obtain the indirect loss functions of each authority :

$$(4)\quad V^M_t(d_t) = 1/2 \underset{m_t}{\text{Min}} (m^2_t + \tau d^2_t) + \alpha V^M_{t+1}(d_{t+1})$$

$$(5)\quad V^F_t(d_t) = 1/2 \underset{f_t}{\text{Min}} (f^2_t + \Gamma d^2_t) + \beta V^F_{t+1}(d_{t+1})$$

We now set initial and terminal conditions :

(6a) d_1 is given

(6b) $\delta V^M_3(d_3)/\ \delta d_3 = \tau d_3$

(6c) $\delta V^F_3(d_3)/\ \delta d_3 = \Gamma d_3$

By backward recursion we get the closed loop Nash strategies for both authorities (see analytical resolution in the appendix) :

$$(7)\quad \{m^c_1, f^c_1\} = \left\{ r\alpha\tau\left(\frac{\Omega^2 + \alpha r^2(1+\alpha\tau)}{x}\right)d_1 \ ,\ -\beta\Gamma r\left(\frac{\Omega^2 + \beta r^2(1+\beta\Gamma)}{x}\right) d_1 \right\}$$

$$(8)\quad \{m^c_2, f^c_2\} = \left\{ (\alpha\tau r/\Omega)d_2 \ ,\ -\ (\beta\Gamma r/\Omega)d_2 \right\}$$

where the superscript "c" stands for closed loop Nash equilibrium.

with $\Omega = 1 + \alpha\tau + \beta\Gamma$

and $x = \Omega^3 + r^2[\alpha^2\tau(1+\alpha\tau) + \beta^2\Gamma(1+\beta\Gamma)]$

From these strategies we get the evolution of the public debt :

(9) $(d^e_1, d^e_2, d^e_3) = (d_1, (r\Omega^2/x)d_1, (r^2\Omega/x)d_1)$

The analysis of the closed loop Nash equilibrium yields several conclusions :

i) The existence of a stationary equilibrium is usually associated with the condition that r is less than one. This means that the real rate of interest is inferior to the growth rate of real income. When monetary and fiscal policies are set by two independent authorities, the condition for a stationary equilibrium is assured if :

(10) $r < 1 + \beta\Gamma + \alpha\tau$

ii) When condition (10) holds, then the closed loop Nash equilibrium strategies are time consistent. Thus, in the absence of precommitment on the part of both monetary and fiscal authorities, these non-cooperative strategies are credible.

iii) When the central bank is independent of the Treasury, the burden of the reduction of public debt is mainly sustained by the Treasury. The lower τ, the slower the rate of debt reduction and the bigger the burden of adjustment placed on the Treasury. Thus when the central bank is completly independent ($\tau = 0$), the monetary target is fulfilled to the detriment of the debt reduction. Conversely, when the fiscal authorities give priority to the deficit, the adjustment of the debt is sustained by the monetary authorities.

iv) When the monetary authorities have a high time preference for the present ($\alpha \to 0$) the burden of debt reduction weighs on the Treasury. As a matter of fact, the cost of debt reduction (3) is higher today than tomorrow for the central bank and it lets Treasury ensure the debt reduction. When both authorities have high time preferences for the present the

(3) Which is a cost of adjustment.

reduction of the public debt will take place only if the real rate of interest is lower than the real growth rate. In this case, both authorities will achieve their own policy goals to the detriment of the reduction of the public debt.

v) When the initial stock of debt is nil, both authorities are at their bliss point.

Thus when condition (10) holds, and even with non-cooperative behavior, there is a progressive reduction of the debt. We will see now if other institutional settings modify the rate of debt reduction.

1.3. Cooperative Equilibrium

Monetary and fiscal policies are now under the aegis of one authority only. This single policymaker can be thought as the parliament or some council whose members come from the central bank and from the Treasury.

It will merge the loss functions of the two separated authorities into a single function which yields :

$$(11)\quad V^s_t(d_t) = 1/2 \min_{f_t, m_t} [f^2_t + \mu m^2_t + (\Gamma + \mu\tau) d^2_t] + \delta V^s_{t+1}(d_{t+1})$$

with $\delta > 0$, $\mu > 0$ and superscript "s" stands for single.

The original central bank objectives relative to those of the Treasury are balanced by μ. δ is time preference for the present for the single authority which solves the following optimal control problem :

$$(12)\quad \min_{m_t, f_t} 1/2 \left[\sum_{t=1}^{T} \delta^t (f_t + \mu m_t)^2 + (\Gamma + \mu\tau) d^2_t + \delta^{T+1}(\Gamma + \mu\tau) d^2_{T+1}\right]$$

s.t. $d_{t+1} = r d_t + f_t - m_t$

Initial and terminal conditions are given by :

(13a) d_1 is given as the initial stock of public debt

(13b) $p^s_3 = (\Gamma + \mu\tau)d_3$

where p^s_3 is the shadow cost of d_3.

The solutions of this optimal control problem give the values of monetary and fiscal policies run by the single authority :

$$(14)\quad \{m^s_1,\ f^s_1\} = \left\{\left[(\Gamma/\mu)+\tau\right]\left[\frac{r(1+\Phi+r^2\delta)}{(1+\Phi)^2+r^2\Phi\delta}\right]d_1,\ -(\Gamma+\mu\tau)\left[\frac{r(1+\Phi+r^2\delta)}{(1+\Phi)^2+r^2\Phi\delta}\right]d_1\right\}$$

$$(15)\quad \{m^s_2,\ f^s_2\} = \left\{\left[\frac{(\Gamma/\mu)+\tau}{(1+\Phi)}\right]r\delta d_1,\ -\left[\frac{(\Gamma+\mu\tau)}{(1+\Phi)}\right]r\delta d_1\right\}$$

where $\Phi = [\Gamma + \mu\tau]\ [1 + (1/\mu)]\delta$

In the cooperative equilibrium the evolution of the public debt is then :

$$(16)\quad (d^s_1, d^s_2, d^s_3) = \left(d_1,\ \frac{r(1+\Phi)}{(1+\Phi)^2 + r^2\Phi\delta}d_1,\ \frac{r^2}{(1+\Phi)^2 + r^2\Phi\delta}d_1\right)$$

Let us now compare the monetary and fiscal policies run by two decentralized authorities to those implemented by a single controller. According to equation (9) and (16), coordination increases the rate of public debt reduction if the following condition holds :

$$(17)\quad \frac{(\beta\Gamma + \alpha\tau)\mu}{(1+\mu)(\Gamma+\mu\tau)} < \delta$$

This condition implies that the rate of public debt reduction is not always higher with a single controller than with two authorities, i.e. coordination does not always pay. This result depends on the value of μ (the weight of the objectives of the central bank in the overall objectives of the single policymaker) and of δ the time preference for the present of the single controller. When it neglects the objectives of the central bank (μ nil), condition (17) is always fulfilled and the fiscal authorities are at their bliss point. Conversely when the single controller ignores the objectives of the Treasury (μ tends to infinity), condition (17) still holds and the monetary authorities are at their bliss point.

Figure 1 represents the static game between monetary and fiscal authorities. The line mm (resp ff) is the reaction function of the monetary authorities (resp fiscal authorities). These two lines intersect at the Nash equilibrium point. Point M (resp F) is the bliss point of the monetary authorities (resp fiscal authorities). The dotted curve represents cooperative equilibria with respect to the value of μ. When μ is in the neighborhood of zero, the cooperative equilibrium is in the neighborhood of F. Conversely when μ tends to infinity, the cooperative equilibrium is in the neighborhood of M.

Lastly when both the central bank, the Treasury and the single authority have the same time preference for the present, condition (17) is always verified. Consequently and without ambiguity, the rate of public debt reduction is always higher when policies are implemented by a single controller.

1.4. Monetary and fiscal strategies with precommitment

Cooperative institutional arrangments are not always feasible. However even if monetary and fiscal policies remain decentralized between two distinct authorities, a better equilibrium than the closed loop Nash can be reached. It is the case when each authority precommits itself on a sequence of moves. The central bank thus determines its own sequence of moves with the knowledge of the moves done by the Treasury. The sequence of moves to which it commits itself is the optimal response to the sequence of moves chosen by the Treasury. The same conditions are required for the Treasury. The equilibrium obtained is thus an open loop Nash equilibrium derived from a two-period model. The Hamiltonians for each authorities are the followings :

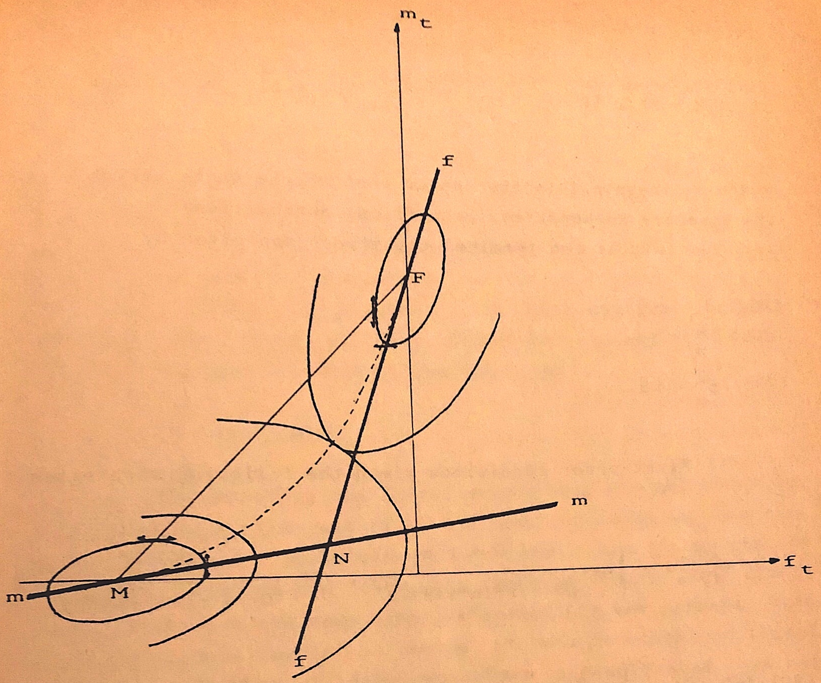

figure 1

(18) $H^M_t = \alpha^t/2\,[m^2_t + \tau d^2_t] + \alpha^{t+1}p^M_{t+1}[rd_t + f_t - m_t - d_{t+1}]$

(19) $H^F_t = \beta^t/2\,[f^2_t + \Gamma d^2_t] + \beta^{t+1}p^F_{t+1}[rd_t + f_t - m_t - d_{t+1}]$

where p^M_t (resp p^F_t) is the shadow cost of the state variable for the monetary authorities (resp fiscal authorities).

Initial and terminal conditions are given by :

(20a) d_1 is given

(20b) $p^M_3 = \tau d_3$

(20c) $p^F_3 = \Gamma d_3$

First order conditions yield the following strategies (4) :

(21) $$\{m^o_1, f^o_1\} = \left\{\alpha\tau r\frac{\Omega + r^2\alpha}{\Omega^2 + r^2(\alpha^2\tau+\beta^2\Gamma)}d_1,\ -\beta\Gamma r\frac{\Omega + r^2\beta}{\Omega^2 + r^2(\alpha^2\tau+\beta^2\Gamma)}d_1\right\}$$

(22) $$\{m^o_2, f^o_2\} = \left\{\frac{\alpha\tau r^2}{\Omega^2 + r^2(\alpha^2\tau+\beta^2\Gamma)}d_1,\ \frac{-\beta\Gamma r^2}{\Omega^2 + r^2(\alpha^2\tau+\beta^2\Gamma)}d_1\right\}$$

The superscript "o" in equations (21), (22) and (23) stands for open loop.

To these open loop equilibrium strategies is associated the following time path of the public debt :

(23) $$(d^o_1, d^o_2, d^o_3) = \left(d_1,\ \frac{r\Omega}{\Omega^2 + r^2(\alpha^2\tau+\beta^2\Gamma)}d_1,\ \frac{r^2}{\Omega^2 + r^2(\alpha^2\tau+\beta^2\Gamma)}d_1\right)$$

When monetary and fiscal authorities act in an open

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(4) See appendix for the complete analytical solutions.

loop framework, the rate of debt reduction is higher than when they act in a closed loop framework. This can be seen from equations (9) and (23). Without ambiguity we can conclude that $d^o_3 < d^c_3$.

The comparison with the cooperative equilibrium strategies is ambiguous and we cannot conclude a priori as to the superiority of one equilibrium over the other. As it will be shown in the next part, we have made numerical simulations in order to ease the comparison of the different kinds of equilibria. Table 3 allows us to conclude that without ambiguity the rate of debt reduction is higher in the cooperative framework than in the open loop.

1.5. Conclusion

When monetary and fiscal authorities are not bound by a precommitment, the rate of public debt reduction is moderate. Under different institutional settings this rate can be increased. This is the case when both monetary and fiscal authorities have the same time preference for the present. When a single controller is in charge of both monetary and fiscal policies, the rate of debt reduction is accelerated. We can draw similar conclusions when each authority makes a precommitment as regards its own strategy. The reduction is accelerated vis à vis the closed loop but not vis à vis the cooperative equilibrium.

II. Fiscal Authorities, Monetary Authorities and the Private Sector

The strategic behavior of both authorities and the link between targets and instruments of the economic policy are modified as soon as we introduce the private sector in the model. Private sector is characterized by its demand for money and by its anticipation behavior in one simplified model of the economy. It has no strategic behavior, so it will not directly interact in the game between fiscal and monetary authorities. Nevertheless by its anticipation behavior, private sector put an additional constraint on the strategic choices of both authorities. This will change the value of the equilibria we obtained in the first part.

2.1. The new strategic model

The economy is characterized by three equations :

(24) $y_t = h(x_t - x^e_t)$, with $x_t = p_t - p_{t-1}$

(25) $y_t = e(l_t - p_t) + g_t$ with $x^e_t = p^e_t - p_{t-1}$

(26) $x^e_t - x^e_{t-1} = (1 - d)(x_{t-1} - x^e_{t-1})$

All variables are in deviation from their stationary level. y denotes the difference between real output and real output at its natural level, p the general price level, x the current rate of inflation, x^e the anticipated rate of inflation, l the nominal stock of money, and g is the budget deficit. h, d, and e are positive parameters.

Equation (24) is an aggregate supply function and (25) an aggregate demand function with a real balance effect. Equation (26) describes an adaptative expectations behavior. From (24) and (25) we have :

(27) $y_t - y_{t-1} = h\,[(x_t - x_{t-1}) - (x^e_t - x^e_{t-1})]$
(28) $y_t - y_{t-1} = e(\theta_t - x_t)$
where θ is the rate of monetary growth.

From (24) and (26) we get :

(29) $x^e_t - x^e_{t-1} = [(1-d)/h]y_{t-1}$

With (28) and (29) in (27) we get :

(30) $x_t = [e/(h+e)]\theta_t + [h/(h+e)]x_{t-1} + [(1-d)/(h+e)]y_{t-1}$

By assumption the creation of monetary base against liabilities of the Treasury is equal to the growth of the stock of money. Since aggregate demand depends on public expenditures, then by (30) inflation in period t is positively related to inflation in period t-1, to money creation between t-1 and t, and to the level of budget deficit in period t-1.

Using the above notations (30) can be rewritten as :

(31) $\pi_{t+1} = a\pi_t + bf_t + cm_t$

where π is the rate of inflation per output unit.

According to equation (31) the current choice of monetary and fiscal instruments influence the motion of future inflation. For each policymaker the relationship between its instruments and its ultimate macroeconomic target is not invariant to changes in the instrument of the other policymaker. Thus the inflation rate is explicitely introduced in the loss function of both monetary and fiscal authorities :

(32) $$L^M = 1/2 \left[\sum_{t=1}^{T} \alpha^t \left(m_t^2 + \sigma\pi_t^2 + \tau d_t^2\right) + \alpha^{T+1}\left(\sigma\pi_{T+1}^2 + \tau d_{T+1}^2\right) \right]$$

(33) $$L^F = 1/2 \left[\sum_{t=1}^{T} \beta^t \left(f_t^2 + \phi\pi_t^2 + \Gamma d_t^2\right) + \beta^{T+1}\left(\phi\pi_{T+1}^2 + \Gamma d_{T+1}^2\right) \right]$$

where σ and ϕ are the weight of inflation for the central bank and the Treasury respectively.

We make the further assumption that the target inflation of both authorities is zero. Including two different targets π^M and π^F would have simply added inertia in the solution of each equilibrium.

The loss functions (32) and (33) with the constraints (1) and (31) define the new strategic model (5).

As above, the various equilibria of the dynamic game are derived from a two-period model. However the analytical

(5) If the targets $\bar{f}$, $\bar{m}$, d^F, d^M, π^F and π^M are equal to zero at their stationary level, the budget constraint is not modified if it is measured in deviation from its stationary level.

solutions are more cumbersome impeding any superficial comparison between the different equilibria. We have thus proceeded to a numerical simulation (6) in order to rank them.

2.2. Closed Loop Nash, Open Loop Nash and Cooperative equilibria

The central bank chooses a closed loop rule for money creation in which each move depends on the current states of inflation and public debt taking as given the closed loop rule for budget deficit. Therefore the central bank assumes that the government will choose the optimal current state feedback rule. The government plays the same way. The closed loop Nash equilibrium strategies are computed with dynamic programming methods since they are based on current state variables. Hence the closed loop Nash equilibrium is dynamically consistent (Analytical resolution is given by equations (A10), (A11), (A18) and (A19) in the appendix).

Consider now the open loop Nash equilibrium. Both decentralized authorities choose their respective sequences of moves. But the central bank and the government now precommit themselves to make their moves without attempting to coordinate their strategies. Hence each authority precommits to the sequence of moves which is the optimal response to the sequence of moves announced by its opponent. These open loop Nash strategies are computed by means of optimal control methods (see analytical resolution in the appendix). With no binding commitment the open loop strategies are generally not credible.

Finally when a single policymaker implements both monetary and fiscal policies it weights the original central bank objective relative to those of the government. The dynamic game between two decentralized authorities degenerates into an optimal control problem. The optimization problem solved by the single controller then leads to a cooperative equilibrium.

For all the equilibria, we have computed a benchmark simulation. In this benchmark simulation parameters σ, ϕ, Γ, τ, β, α, δ, μ and r are equal to unity and parameters a, b, c are

(6) The analytical solutions are described in the appendix.

0.6, 0.1 and 0.3 respectively. Each strategy is given as a function of state variables π_1 and d_1. We have change the value of each parameter in order t、 compare the various strategies within the same equilibrium concept. These simulations are set out in Table I.

2.2.1. General properties

i) The relationship between inherited inflation and both monetary and fiscal strategies is negative whatever the values of the parameters are and for each kind of equilibrium. For the monetary authority, high inflation involves a contractionary policy and for fiscal authority, high inflation involves a weak monetization of the public debt. Hence the fiscal authorities will have to come up with a sizeable budget surplus.

ii) Whatever the value of the parameters and whichever equilibrium concept is used, the inherited public debt has a positive impact on monetary strategies and a negative impact on fiscal strategies. Ceteris paribus, the higher the inherited debt, the greater its monetization by the central bank and the higher the budget surplus to be achieved.

iii) In period 1, the existing public debt leads fiscal authorities to generate a budget surplus that is lower in the closed loop Nash equilibrium than in the open loop Nash or in the cooperative equilibrium. In contrast, in period 2, the budget surplus is higher in the closed loop Nash equilibrium than in the two other equilibria. Analogous conclusions can be drawn for the monetary authority in their strategy of debt monetization. The reason is that in closed loop, both authorities plan the reduction of the debt over the two periods. The fiscal strategy (resp monetary strategy) in period 1 takes into account the monetary strategy (resp fiscal strategy) for period 1 and 2, so the reduction of the public debt is clearly assessed all along the two periods. In open loop equilibrium each authority care only about the strategy of the other within the same period. So the behavior of each authority is less smooth than in the closed loop Nash.

Table 1

Comparative strategies in closed loop Nash equilibrium, in open loop Nash equilibrium and in cooperative equilibrium

	f_1/d_1	f_2/d_1	f_1/π_1	f_2/π_1	m_1/d_1	m_2/d_1	m_1/π_1	m_2/π_1
benchmark*								
CLN	-0,372	-0,106	-0,172	-0,072	0,332	0,085	-0,248	-0,041
OLN	-0,381	-0,100	-0,129	-0,059	0,342	0,078	-0,158	-0,060
cooperation	-0,505	-0,187	ns	-0,104	0,307	0,059	ns	-0,142
changing the weights of public debt								
$r = 0$								
CLN	-0,025	-0,015	-0,072	-0,030	0,559	0,177	-0,173	-0,012
OLN	-0,025	-0,015	-0,074	-0,030	0,559	0,177	-0,121	-0,040
cooperation	-0,507	-0,111	ns	-0,095	0,176	0,010	0,015	-0,151
$r = 2$								
CLN	-0,534	-0,110	-0,220	-0,080	0,242	0,051	-0,270	-0,053
OLN	-0,533	-0,109	-0,151	-0,008	0,255	0,048	-0,170	-0,068
cooperation	-0,504	-0,072	-0,010	-0,108	0,361	0,040	ns	-0,137
$\tau = 0$								
CLN	-0,580	-0,189	-0,240	-0,100	0,023	0,013	-0,344	-0,075
OLN	-0,580	-0,188	-0,162	-0,071	0,023	0,013	-0,200	-0,083
cooperation	-0,507	-0,111	ns	-0,095	0,176	0,010	0,015	-0,151
$\tau = 2$								
CLN	-0,277	-0,071	-0,141	-0,059	0,490	0,090	-0,190	-0,031
OLN	-0,290	-0,067	-0,115	-0,050	0,444	0,089	-0,136	-0,051
cooperation	-0,504	-0,072	-0,010	-0,108	0,361	0,040	ns	-0,137

*In the Benchmark simulation, the values of the parameters $\sigma, \phi, r, \tau, \beta, \alpha, \delta, \mu$ and r are equal to 1. Moreover $a = 0,6$, $b = 0,1$ and $c = 0,3$.
$x_i/y_j = \partial x_i/\partial y_j$ (see Appendix)

Table 1

Comparative strategies in closed loop Nash equilibrium, in open loop Nash equilibrium and in cooperative equilibrium

	f_1/d_1	f_2/d_1	f_1/π_1	f_2/π_1	m_1/d_1	m_2/d_1	m_1/π_1	m_2/π_1
benchmark								
CLN	-0,372	-0,106	-0,172	-0,072	0,332	0,085	-0,248	-0,041
OLN	-0,381	-0,100	-0,129	-0,059	0,342	0,078	-0,158	-0,060
cooperation	-0,505	-0,087	ns	-0,104	0,307	0,059	ns	-0,142
changing the weights of inflation								
$\sigma = 0$								
CLN	-0,361	-0,100	-0,050	-0,017	0,350	0,094	-0,033	-0,017
OLN	-0,369	-0,094	-0,047	-0,017	0,359	0,088	-0,032	-0,016
cooperation	-0,458	-0,080	ns	-0,056	0,361	0,061	ns	-0,076
$\sigma = 2$								
CLN	-0,382	-0,112	-0,274	-0,115	0,318	0,079	-0,427	-0,052
OLN	-0,390	-0,106	-0,197	-0,092	0,329	0,073	-0,263	-0,092
cooperation	-0,554	-0,090	ns	-0,145	0,248	0,064	0,017	-0,200
$\phi = 0$								
CLN	-0,367	-0,104	-0,132	-0,060	0,336	0,088	-0,223	-0,029
OLN	-0,376	-0,098	-0,089	-0,045	0,346	0,081	-0,132	-0,048
cooperation	-0,458	-0,080	ns	-0,056	0,361	0,061	ns	-0,076
$\phi = 2$								
CLN	-0,377	-0,109	-0,210	-0,084	0,329	0,083	-0,271	-0,052
OLN	-0,386	-0,103	-0,168	-0,071	0,339	0,077	-0,182	-0,072
cooperation	-0,554	-0,090	ns	-0,145	0,248	0,061	ns	-0,200

Table 1

Comparative strategies in closed loop Nash equilibrium, in open loop Nash equilibrium and in cooperative equilibrium

	f_1/d_1	f_2/d_1	f_1/π_1	f_2/π_1	m_1/d_1	m_2/d_1	m_1/π_1	m_2/π_1
benchmark								
CLN	-0,372	-0,106	-0,172	-0,072	0,332	0,085	-0,248	-0,041
OLN	-0,381	-0,100	-0,129	-0,059	0,342	0,078	-0,158	-0,060
cooperation	-0,505	-0,087	ns	-0,104	0,307	0,059	ns	-0,142
changing time preferences								
β = 0,9								
CLN	-0,344	-0,104	-0,158	-0,069	0,349	0,092	-0,239	-0,037
OLN	-0,352	-0,098	-0,125	-0,055	0,359	0,085	-0,155	-0,058
α = 0,9								
CLN	-0,387	-0,112	-0,163	-0,069	0,308	0,084	-0,231	-0,041
OLN	-0,396	-0,106	-0,123	-0,056	0,317	0,078	-0,147	-0,058
δ = 0,9								
cooperation	-0,492	-0,091	ns	-0,094	0,303	0,066	ns	-0,130
σ = 0, τ = 0								
CLN	-0,590	-0,198	-0,025	ns	0	0	0	0
OLN	-0,590	-0,198	-0,025	ns	0	0	0	0
cooperation	-0,470	-0,120	ns	-0,050	0,313	0,102	ns	-0,081
the specialization of authorities σ = 2 ϕ = 0 τ = 0 r = 2								
CLN	-0,687	-0,160	-0,451	-0,167	0,047	0,024	-0,595	-0,106
OLN	-0,688	-0,159	-0,285	-0,138	0,047	0,024	-0,358	-0,134

Table 1

Comparative strategies in closed loop Nash equilibrium, in open loop Nash equilibrium and in cooperative equilibrium

	f_1/d_1	f_2/d_1	f_1/π_1	f_2/π_1	m_1/d_1	m_2/d_1	m_1/π_1	m_2/π_1
benchmark								
CLN	-0,372	-0,106	-0,172	-0,072	0,332	0,085	-0,248	-0,041
OLN	-0,381	-0,100	-0,129	-0,059	0,342	0,078	-0,158	-0,060
cooperation	-0,505	-0,087	ns	-0,104	0,307	0,059	ns	-0,142
changing the other parameters								
r = 0,9								
CLN	-0,331	-0,089	-0,154	-0,068	0,296	0,070	-0,227	-0,046
OLN	-0,338	-0,084	-0,128	-0,059	0,304	0,060	-0,150	-0,060
cooperation	-0,454	-0,073	ns	-0,105	0,271	0,049	ns	-0,141
r = 1,1								
CLN	-0,414	-0,125	-0,185	-0,077	0,370	0,101	-0,270	-0,035
OLN	-0,425	-0,117	-0,130	-0,059	0,382	0,093	-0,157	-0,060
cooperation	-0,555	-0,102	-0,010	-0,102	0,344	0,070	ns	-0,143
b = 0								
CLN	-0,373	-0,108	-0,135	-0,061	0,326	0,083	-0,227	-0,031
OLN	-0,382	-0,101	-0,090	-0,040	0,337	0,076	-0,134	-0,049
cooperation	-0,482	-0,095	-0,014	-0,067	0,324	0,051	0,011	-0,127

The cooperative equilibrium is characterized by a reduction of the stock of public debt in period 1 and by a reduction of inflation in period 2. Thus the strategy of the policymaker is to provide a high budget surplus in period 1 and a tight monetary policy in period 2.

Let us now compare the strategies when we modify the weight of public debt and of inflation in the objectives of both authorities.

2.2.2. Changing the weights of public debt and inflation

- Changing the weights of public debt (Γ, τ)

A variation in the value of the debt parameters (Γ, τ) has a more significant effect on the closed loop and open loop Nash equilibrium strategies than on the cooperative equilibrium strategies (7).

Strategies of a given authority are more reactive to a variation in its own parameter than to a variation in the parameter of the other authority. Thus when fiscal authorities modify the weight of the public debt in their objective function, their own strategies change much more than monetary strategies. For fiscal authorities, the higher the weight assigned to the debt, the higher the budget surplus and the lower money creation. Conversely for the monetary authorities the higher the weight assigned to the debt, the greater the monetary creation and the lower the budget surplus.

When Γ or τ are nil, the closed loop Nash and open loop Nash strategies are the same.

- Changing the weights of inflation (σ, ϕ)

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(7) The reason is that in cooperative equilibrium the weight assigned to the debt is ($\Gamma + \mu\tau$). Thus a variation of Γ or of τ has only small effects on the value of that weight hence on the strategy.

A variation of the weight assigned to inflation has no significant effects on the strategy of debt reduction in closed loop Nash and in open loop Nash, whatever the authority is. Nonetheless the variability of monetary or fiscal strategies is greater when σ varies than when ø varies. As a matter of fact, a variation of σ (weight of inflation in the central bank objective function) has an immediate effect on the monetization of the debt when ø has no direct effect on the monetization. In a cooperative equilibrium the situation will be more sensitive to changing the weights of inflation than when the weight of the debt was modified. As was explained above, this is a result of the general strategy in cooperative equilibrium, characterized by a tight monetary policy in the second period in order to fight inflation.

2.2.3. Changing the time preference

When one of the two authorities has a higher time preference for the present the cost of adjustment is sustained by the other one. When the fiscal authorities has a higher time preference for the present (β is lower) the budget surplus is lower in both periods. In return the amount of monetary creation rises, the central bank bearing the burden of the adjustment. Conversely when α decreases, money creation decreases and the adjustment occurs through the budget surplus. In cooperative equilibrium a greater time preference implies a smoother evolution. When a single policymaker implements both monetary and fiscal policies the adjustment occurs over time. Compared to the benchmark simulation, the fiscal surplus and the money creation are lower in period 1 and higher in period 2.

2.3. Debt evolution in Closed Loop Nash Equilibrium and in the Cooperative Open Loop Nash Equilibrium and in the Cooperative Equilibrium

The equilibrium strategies shape the evolution over time of the public debt. As in the preceding section we have made simulations in order to rank them by effectiveness. All these simulations are compared with the benchmark simulation with the same parameters as in the preceding section. (see Table 2)

2.3.1. General properties

i) For all values of the parameters, the public debt reduction is always fastest in the cooperative equilibrium, then in the open loop Nash equilibrium and, finally, in the closed loop Nash equilibrium.

ii) For all the simulations the inherited public debt has more influence on the time path of the debt than the initial inflation. Conversely inherited inflation has a greater impact on the evolution of inflation than the initial public debt.

2.3.2. Changing the weights of public debt and inflation

- Changing the weights of public debt (Γ, τ)

The more both authorities care about public debt reduction the faster the stock of public debt decreases. However the time path of public debt is more sensitive to variations in Γ (Treasury) than to variations in τ (central bank). (see Figure 2a and 2b)

Whatever the equilibrium concept is :

i) When τ is less than one, the central bank pays more attention to its target of inflation. So the monetization of the debt is low and this in turn contrains the fiscal authorities to have a tighter policy. When Γ is less than one, the Treasury neglects its public debt target and concerns itself primarily with its objectives of fiscal stimulus. A substantial reduction of the debt occurs only through the monetization behavior of the central bank.

Thus, the stock of the public debt is lower with a low τ than with a low Γ.

Table 2

The comparative evolution of public debt in the open loop Nash equilibrium, in the closed loop Nash equilibrium and in the cooperative equilibrium

	d_2/d_1	d_3/d_1	d_2/π_1	d_3/π_1
benchmark				
CLN	0,294	0,101	0,075	0,044
OLN	0,276	0,045	0,028	0,029
cooperation	0,187	0,040	−0,016	0,021
	changing the weights of inflation			
	$\sigma = 0$			
CLN	0,288	0,094	−0,016	−0,017
OLN	0,270	0,081	−0,015	−0,016
cooperation	0,179	0,037	ns	0,011
	$\sigma = 2$			
CLN	0,299	0,107	0,153	0,091
OLN	0,280	0,101	0,065	0,066
cooperation	0,197	0,042	−0,024	0,029
	$\phi = 0$			
CLN	0,296	0,106	0,090	0,060
OLN	0,277	0,098	0,043	0,045
cooperation	0,179	0,037	ns	0,011
	$\phi = 2$			
CLN	0,292	0,099	0,060	0,029
OLN	0,274	0,093	0,013	0,014
cooperation	0,197	0,042	−0,024	0,029
	changing the weights of public debt			
	$r = 0$			
CLN	0,415	0,222	0,101	0,080
OLN	0,414	0,222	0,047	0,050
cooperation	0,316	0,108	−0,023	0,033
	$r = 2$			
CLN	0,223	0,056	0,057	0,027
OLN	0,211	0,053	0,020	0,019
cooperation	0,134	0,021	−0,010	0,015
	$\tau = 0$			
CLN	0,396	0,193	0,103	0,076
OLN	0,395	0,193	0,043	0,049
cooperation	0,316	0,108	−0,023	0,033
	$\tau = 2$			
CLN	0,228	0,061	0,056	0,029
OLN	0,214	0,058	0,021	0,020
cooperation	0,134	0,021	−0,010	0,015

Table 2

The comparative evolution of public debt in closed loop Nash equilibrium, in open loop Nash equilibrium, and in cooperative equilibrium

	d_2/d_1	d_3/d_1	d_2/π_1	d_3/π_1
benchmark				
CLN	0,294	0,101	0,075	0,044
OLN	0,276	0,045	0,028	0,029
cooperation	0,187	0,040	-0,016	0,021
	changing the time preferences			
	$\beta = 0,9$			
CLN	0,306	0,109	0,080	0,048
OLN	0,288	0,103	0,029	0,032
	$\alpha = 0,9$			
CLN	0,303	0,107	0,067	0,040
OLN	0,286	0,101	0,024	0,026
	$\delta = 0,9$			
cooperation	0,204	0,047	-0,014	0,021
	$\sigma = 0,\ \tau = 0$			
CLN	0,402	0,204	-0,025	-0,030
OLN	0,402	0,204	-0,025	-0,030
cooperation	0,308	0,116	-0,012	0,170
	specialization of the authorities			
	$\sigma = 2,\ \phi = 0,\ \tau = 0,\ \Gamma = 2$			
CLN	0,265	0,080	0,144	0,083
OLN	0,264	0,079	0,073	0,069
	changing the other parameters			
	$r = 0,9$			
CLN	0,270	0,084	0,067	0,039
OLN	0,257	0,080	0,030	0,029
cooperation	0,174	0,033	-0,016	0,020
	$r = 1,1$			
CLN	0,315	0,119	0,082	0,049
OLN	0,290	0,111	0,026	0,029
cooperation	0,199	0,047	-0,017	0,020
	$b = 0$			
CLN	0,299	0,108	0,092	0,061
OLN	0,280	0,101	0,044	0,046
cooperation	0,193	0,017	-0,026	0,033

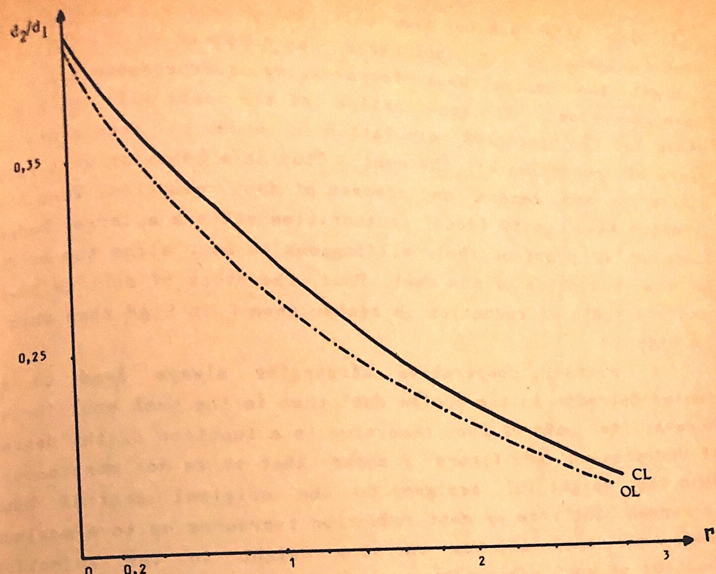


Figure 2a

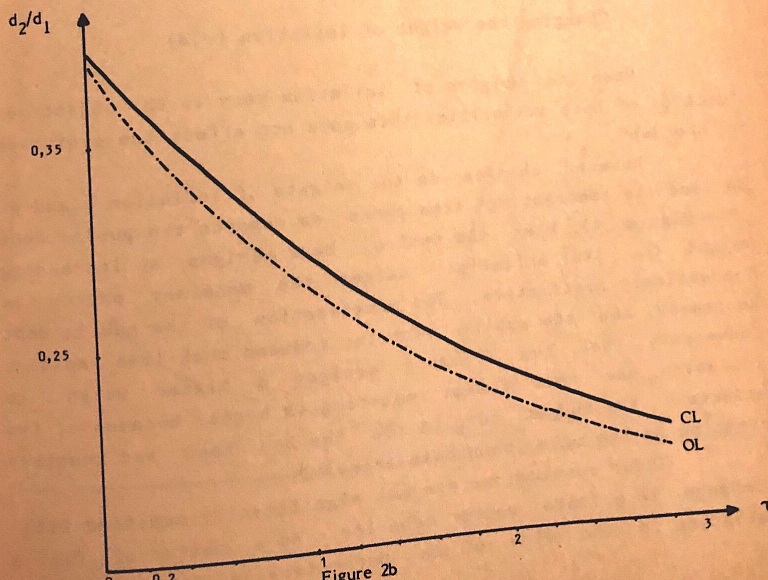


Figure 2b

ii) When τ is greater than unity the monetary authority gives more weights to its debt target as compared to its inflation target. The central bank forsakes its independence vis à vis the government. The monetization of the debt will be higher than in the benchmark simulation in order to accelerate the rate of reduction of the debt. Thus this behavior will raise inflation and lessen the process of debt reduction. When Γ is greater than unity fiscal authorities achieve a larger budget surplus, a proof of their willingness to bear alone the burden of the reduction of the debt. Thus, the stock of debt is lower and its rate of reduction is higher when Γ is high than when τ is high

Finally, cooperative strategies always lead to a faster decrease in the public debt than in the Nash equilibria. However the rate of debt reduction is a function of the degree of cooperation and figure 3 shows that it is not monotonous. When the weight (μ) assigned to the original central bank increases, the rate of debt reduction increases up to a maximum where each authority has the same weight in the objective function of each policymaker, and decreases thereafter.

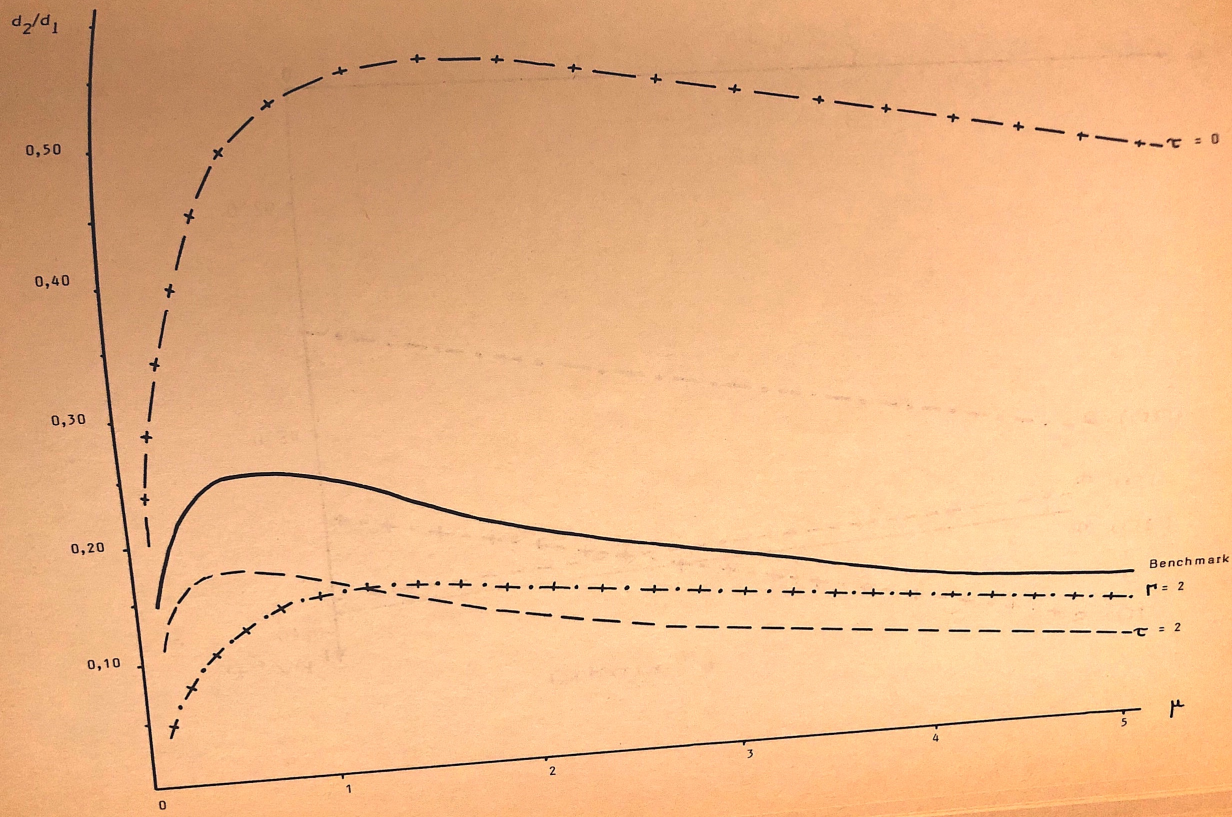
- Changing the weight of inflation (σ, ϕ)

When the weights of inflation vary in the objective function of both authorities this does not affect the evolution of the debt.

However, changes in the weights of inflation σ and ϕ do lead to contrasting time paths as regards the public debt (see Figure 4). When the central bank assigns an increasing weight to its inflation target the monetary policy is increasingly restrictive. The monetization of the public debt decreases, and the public debt is reduced much less rapidly. Conversely, when the Treasury assigns a higher weight to inflation, the rate of debt reduction is higher because of two effects : the budget surplus, on the one hand, and monetary creation on the other hand both increase.

These conclusions are not significantly modified with a change in private sector behavior. As a matter of fact a variation in the value of the parameters in the equation of

Figure 3



d_2/d_1
0,50
0,40
0,30
0,20
0,10
0
1
2
3
4
5
μ
τ = 0
Benchmark
Γ = 2
τ = 2

figure 4

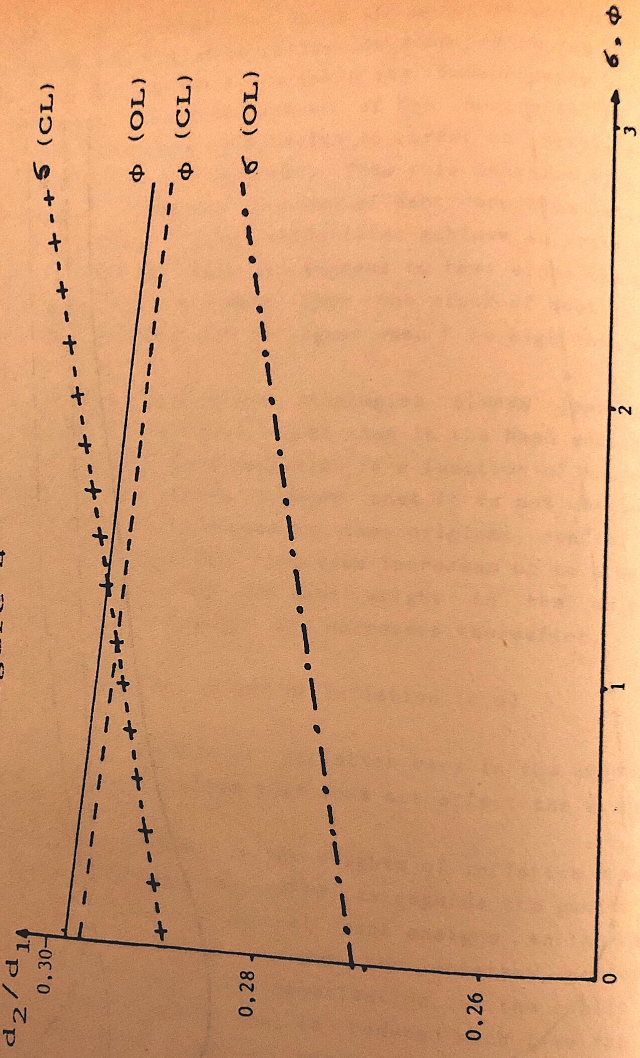
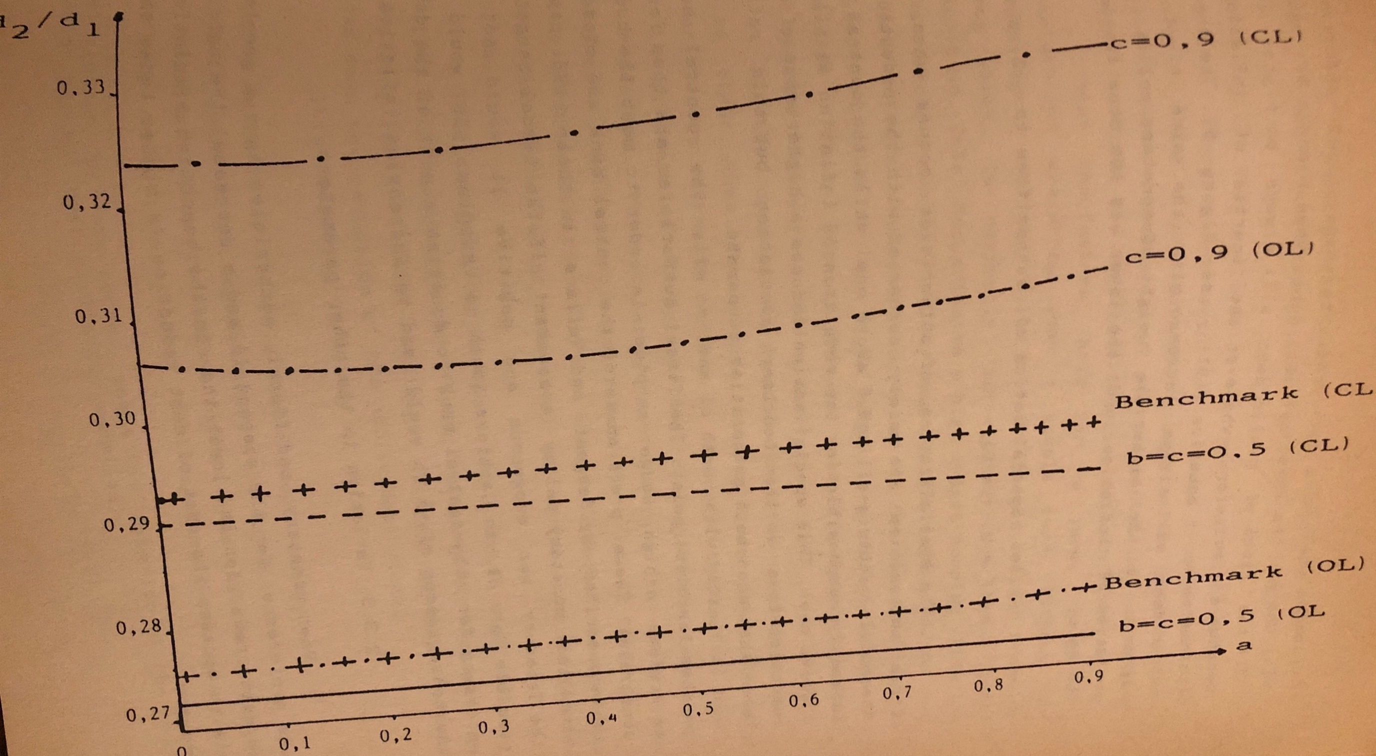


figure 5



d2/d1
0,33
0,32
0,31
0,30
0,29
0,28
0,27
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
a
c=0,9 (CL)
c=0,9 (OL)
Benchmark (CL)
b=c=0.5 (CL)
Benchmark (OL)
b=c=0,5 (OL

inflation (31) will not modify the rate of debt reduction, it will change the level of the debt. When b and c are equal in (31) the stock of public debt will always be higher in the benchmark simulation, whatever the inertia of inflation is (parameter a in equation (31) - see Figure 5). The stock of public debt will always increase with the value of c. The higher c is, the higher the weight of monetary policy in the evolution of inflation, and the lower is the monetization of the debt.

- The Specialization of authorities ($\sigma = 2$, $\phi = 0$, $\Gamma = 2$, $\tau = 0$)

The specialization of authorities occurs when the fiscal authorities are mainly concerned with the reduction of the public debt (ϕ nil and Γ above one) while the central bank is mainly responsible for the reduction of inflation (τ nil and σ above one). This specialisation induces a higher rate of debt reduction than in the benchmark simulation but this rate is lower than when both authorities cooperate.

Lastly with both σ and τ nil, the central bank reaches its bliss point. The fiscal authorities are thus forced to achieve a high budget surplus in order to meet the budget constraint. These preferences of the central bank are close to those described by Sargent and Wallace in their 1981 paper, since the monetary policy is constant. If the fiscal target of the Treasury was exogenous and positive it would not be possible for it to achieve such a surplus. It would be necessary for the central bank to monetize a part of the debt, the consequences of which would lead to increasing inflation.

2.3.3. Variation in the other parameters

When monetary and fiscal authorities have a greater time preference for the present (β and α decrease) the rate of the debt reduction is lower than in the benchmark simulation. This is because the cost of debt reduction is higher today than tomorrow.

2.4. Comparing the evolution of the public debt with or without the private sector

For the Nash equilibria, the reduction of the debt is faster when the private sector is excluded from the analysis (see Table 3 and Figure 6). This comes from the supplementary constraint that the behavior of private sector introduces into the model and into the objective function of both authorities.

These conclusions hold for a large range of parameters but are wrong when τ equals zero. Without the private sector (8) the value τ = 0 implies a tight monetary policy which is correctly anticipated by the fiscal authorities. This necessitates a high budget surplus. With the private sector, monetary policy has to fight inflation, so that the rate of debt reduction increases for two reasons : on the one hand, because of the high budget surplus (which has the same size with or without private sector) and, on the other hand because of the restrictive monetary policy.

In the cooperative equilibrium, the conclusions are not so clear. When the balance between fiscal and monetary authorities is strongly marked in favour of the central bank, then the presence of the private sector accelerates the rate of debt reduction. When Γ or τ are superior to unity, the policymaker's behavior is to reduce debt in the first period and to reduce inflation in the second. Without the private sector the reduction of the debt is carried out over the two periods.

Thus the behavior of the private sector significantly modifies both monetary and fiscal strategies whatever the prevailing institutionnal settings. In general this behavior slows down the reduction of the public debt stock.

This reduction is slower if :

i) both authorities act in a closed loop strategy ;
ii) both authorities neglect their debt objective ;
iii) monetary authorities have inflation as their main target ;
iv) both authorities exhibit a strong time preference for the

---

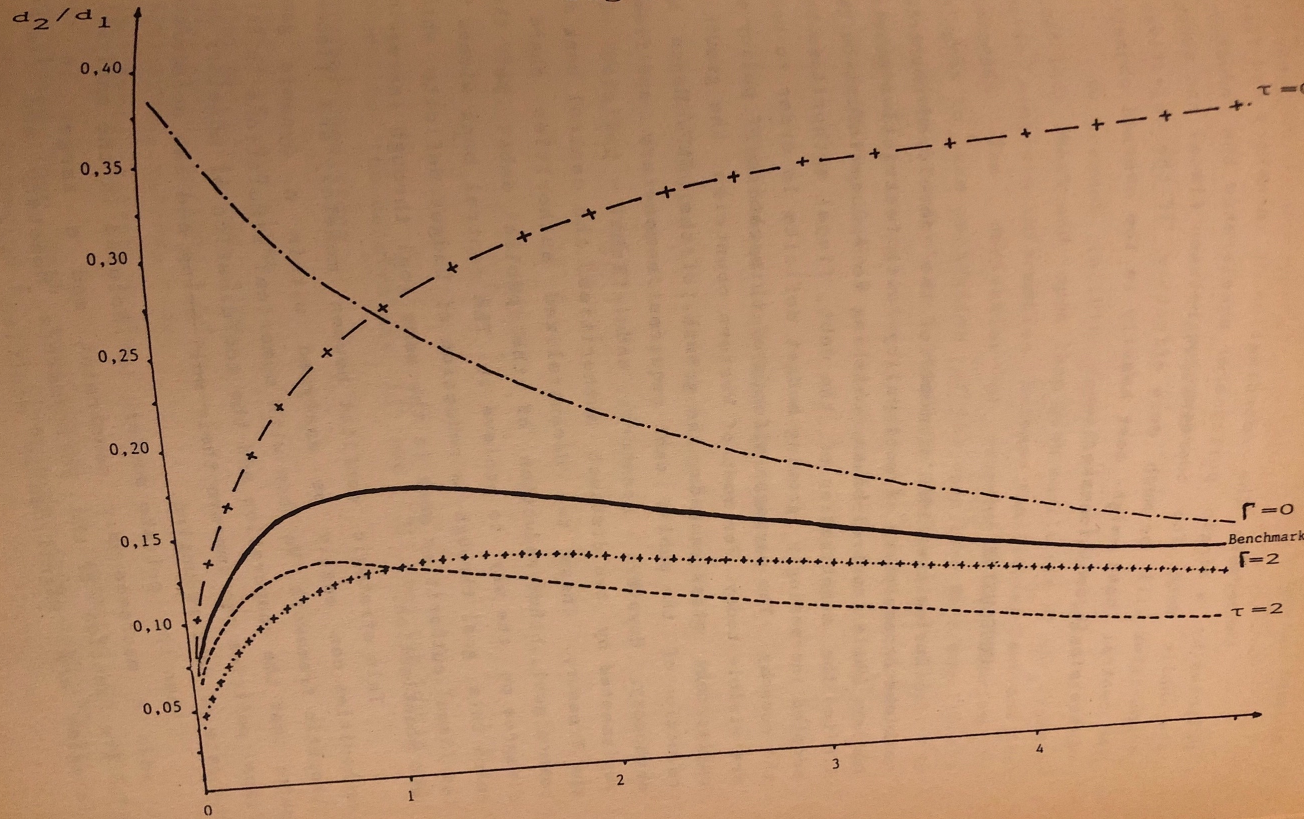
(8) Without private sector means the behavior of the private sector has not been explicitly modelled.

Table 3

The comparative evolution of public debt with and without private sector

	without private sector		with private sector	
	d_2/d_1	d_3/d_1	d_2/d_1	d_3/d_1
CLN				
benchmark	0,290	0,096	0,294	0,101
$\Gamma = 0$	0,400	0,200	0,415	0,222
$\Gamma = 2$	0,222	0,055	0,223	0,056
$\tau = 0$	0,400	0,200	0,396	0,193
$\tau = 2$	0,222	0,055	0,228	0,061
$\beta = 0,9$	0,301	0,103	0,306	0,109
$\alpha = 0,9$	0,301	0,103	0,303	0,107
$r = 0,9$	0,267	0,080	0,271	0,084
OLN				
benchmark	0,272	0,091	0,276	0,095
$\Gamma = 0$	0,400	0,200	0,414	0,222
$\Gamma = 2$	0,210	0,052	0,211	0,053
$\tau = 0$	0,400	0,200	0,395	0,193
$\tau = 2$	0,210	0,052	0,214	0,058
$\beta = 0,9$	0,283	0,097	0,288	0,103
$\alpha = 0,9$	0,283	0,097	0,286	0,101
$r = 0,9$	0,254	0,070	0,257	0,080
S				
benchmark	0,172	0,034	0,187	0,040
$\mu = 0,1$	0,070	ns	0,072	0,036
$\mu = 5$	0,110	0,013	0,106	0,013
$\mu = 20$	0,041	ns	0,034	ns
$\Gamma = 0$	0,272	0,090	0,316	0,108
$\Gamma = 2$	0,200	0,018	0,134	0,021
$\tau = 0$	0,272	0,090	0,316	0,108
$\tau = 2$	0,200	0,018	0,134	0,021
$\delta = 0,9$	0,188	0,041	0,204	0,047
$r = 0,9$	0,159	0,028	0,174	0,033

figure 6


d2/d1
0,40
0,35
0,30
0,25
0,20
0,15
0,10
0,05
0
1
2
3
4
τ =0
Γ =0
Benchmark
Γ=2
τ =2

present.

Conversely, the coordination of monetary and fiscal policies by a single policymaker accelerates the reduction of the public debt. The coordination between fiscal and monetary authorities will be much more effective if the objectives of the central bank weigh most heavily in the overall objectives of the single decisionmaker.

CONCLUSION

During the 1970's, most of the developed countries combined expansionary fiscal policy with restrictive monetary policy. While central banks whishing to reduce inflation have limited the monetization of the debt, fiscal authorities have adopted increasingly growing budget deficits in order to boost the economy. The perverse effects of this choice of policy mix are visible today. For most of Western countries, the growth of public debt stock exceeds the growth of the GNP. Hence the reduction of the public debt requires a monetary and fiscal orthodoxy. However, monetary and fiscal policies are implemented by two distinct authorities, the central bank and the Treasury. These two decentralized authorities share a common goal, the reduction of the public debt, yet they disagree on the way to achieve it. The central bank wishes to reach this goal through a reduction of budget deficits, while the fiscal authorities seek it the same goal through increasing debt monetization.

This strategic conflict beween monetary and fiscal authorities can easily be analyzed within a dynamic game theoretic framework. We show with numerical simulations in this paper that the cooperation and the coordination of monetary and fiscal policies can improve their well-being and can accelerate the rate of debt reduction.

When the private sector is included in the analysis it adds a macroeconomic constraint and a target in the objective function of the policymakers. Monetary and fiscal strategies are significantly modified and the adaptive anticipatory behavior of the private sector slows the rate of debt reduction.

APPENDIX

We present in this appendix the derivation of the closed loop Nash and open loop Nash equilibria of the dynamic game in complete information between the Central Bank and the Government. The derivation of cooperative equilibrium is analogous to the derivation of the open loop Nash equilibrium with a single controller. When both authorities neglect the influence of private sector behavior on their own strategies, the relevant equilibria are computed with σ,ø,a,b and c equal to zero.

1. The closed loop Nash equilibrium

The indirect loss functions of monetary and fiscal authorities are respectively :

$$\text{(A1)}\quad V^M(d_t,\pi_t) = 1/2 \underset{m_t}{\mathrm{Min}}(m_t^2 + \tau d_t^2 + \sigma\pi_t^2) + \alpha V^M(d_{t+1}, \pi_{t+1}),\qquad t = 1,2$$

$$\text{(A2)}\quad V^F(d_t,\pi_t) = 1/2/ \underset{f_t}{\mathrm{Min}}(f_t^2 + \Gamma d_t^2 + \phi\pi_t^2\ ) + \beta V^F(d_{t+1},\pi_{t+1}),\qquad t = 1,2$$

subject to the constraints (1) and (31), and the initial and terminal conditions :

(A3a) d_1, π_1 are given

(A3b) $\delta V^M(d_3,\pi_3)/\delta d_3 = \tau d_3$

(A3c) $\delta V^F(d_3,\pi_3)/\delta d_3 = \Gamma d_3$

(A3d) $\delta V^M(d_3,\pi_3)/\delta \pi_3 = \sigma\pi_3$

(A3e) $\delta V^F(d_3,\pi_3)/\delta \pi_3 = \phi\pi_3$

The closed loop Nash equilibrium is computed by backward recursion. In period t, the first order conditions (FOCs) for a minimum are :

$$\text{(A4)}\quad \delta V^M(d_t,\pi_t)/\delta m_t = 0$$
$$= m_t + \alpha[(\delta V^M(d_{t+1}, \pi_{t+1})/\delta d_{t+1},)(\delta d_{t+1}/\delta m_t)] + \alpha[(\delta V^M(d_{t+1},\pi_{t+1})/\delta \pi_{t+1})(\delta \pi_{t+1}/\delta m_t)]$$

$$\text{(A5)}\quad \delta V^F(d_t,\pi_t)/\delta f_t = 0$$
$$= f_t + \beta[(\delta V^F(d_{t+1},\pi_{t+1})/\delta d_{t+1})(\delta d_{t+1}/\delta f_t)] + \beta[(\delta V^F(d_{t+1},\pi_{t+1})/\delta \pi_{t+1})(\delta \pi_{t+1}/\delta f_t)]$$

1.1. The solution in period 2

Using the FOCs, we obtain :

$$\text{(A6)}\quad m_2 = \alpha(\tau d_3 - \sigma c \pi_3)$$
$$\text{(A7)}\quad f_2 = -\beta(\Gamma d_3 + \o b \pi_3)$$

Given the laws of motion of public debt and inflation, we have :

$$\text{(A8)}\quad m_2 = (M_o d_2 + M_1 f_2 + M_2 \pi_2)/M_3$$
$$\text{(A9)}\quad f_2 = (F_o d_2 + F_1 f_2 + F_2 \pi_2)/F_3$$

with $M_o = \alpha\tau r$, $M_1 = \alpha(\tau - cb\sigma)$, $M_2 = -\alpha\sigma ac$, $M_3 = 1+\alpha(\tau+\sigma c^2)$, $F_o = -\beta\Gamma r$, $F_1 = \beta(\Gamma - cb\o)$, $F_2 = -\beta\o ab$, $F_3 = 1 + \beta(\Gamma + \o b^2)$

We express the closed loop Nash equilibrium strategies in terms of state variables in period 2 :

$$\text{(A10)}\quad m^c_2 = G_o d_2 + G_2 \pi_2$$

$$\text{(A11)}\quad f^c_2 = H_o d_2 + H_2 \pi_2$$

with

$$H_o = (F_o M_3 + F_1 M_o)/(F_3 M_3 - F_1 M_1)$$
$$H_2 = (F_1 M_2 + F_2 M_3)/(F_3 M_3 - F_1 M_1)$$
$$G_o = (M_o F_3 + M_1 F_o)/(F_3 M_3 - F_1 M_1)$$
$$G_2 = (M_1 F_2 + M_2 F_3)/(F_3 M_3 - F_1 M_1)$$

Using these strategies, we derive the motion of public debt and inflation :

(A12) $d_3^c = (r+H_o-G_o)d_2 + (H_2-G_2)\pi_2$

(A13) $\pi_3^c = (bH_o + cG_o)d_2 + (a + bH_2 + cG_2)\pi_2$

and the value functions for period 2 $V^M(d_2,\pi_2)$ and $V^F(d_2,\pi_2)$

1.2. The solution in period 1.

Knowing $V^M(d_2,\pi_2)$ and $V^F(d_2,\pi_2)$, the FOCs for a minimum can be written :

(A14) $m_1 = X_o d_2 + X_2\pi_2$

(A15) $f_1 = Y_o d_2 + Y_2\pi_2$

with

$X_o = \alpha(\tau+\alpha\tau(r+H_o)(r+H_o-G_o) - \alpha\sigma c(\ a+bH_2)(bH_o+cG_o))$

$Y_o = -\beta(\Gamma+\beta\Gamma(r-G_o)(r+H_o-G_o) + \beta\phi b(a+cG_2)(bH_o+cG_o))$

$X_2 = \alpha(\alpha\tau(r+H_o)(H_2-G_2) - c\sigma - \alpha\sigma c(a+bH_2)(a+bH_2+cG_2))$

$Y_2 = -\beta(\beta\Gamma(r-G_o)(H_2-G_2) + c\phi + \beta\phi b(a+cG_2)(a+bH_2+cG_2))$

Substituting (1) and (31) into (A14) and (A15) yields the strategies :

(A16) $m_1 = (P_o d_1 + P_1 f_1 + P_2\pi_1)/P_3$

(A17) $f_1 = (Q_o d_1 + Q_1 f_1 + Q_2\pi_1)/Q_3$

with $P_o = rX_o, P_1 = X_o+bX_2, P_2 = aX_2, P_3 = 1+X_o-cX_2, Q_o = rY_o,$
$Q_1 = Y_o-cY_2, Q_2 = aY_2, Q_3 = 1-Y_o-bY_2$

We express the closed loop Nash strategies in terms of state variables in period 1 :

(A18) $m_1^c = R_o d_1 + R_2\pi_1$

(A19) $f^{c}_{1} = S_o d_1 + S_2 \pi_1$

with

$R_o = (Q_oP_3+Q_1P_o)/(Q_3P_3-Q_1P_1)$

$R_2 = (Q_1P_2+Q_2P_3)/(Q_3P_3-Q_1P_1)$

$S_o = (P_oQ_3+P_1Q_o)/(Q_3P_3-Q_1P_1)$

$S_2 = (P_1Q_2+P_2Q_3)/(Q_3P_3-Q_1P_1)$

Using theses strategies we finally get the motion of public debt and inflation :

(A20) $d^{c}_{2} = (r+S_o-R_o)d_1 + (S_2-R_2)\pi_1$

(A21) $\pi^{c}_{2} = (a+bS_o+cR_o)d_1 + (bS_2+cR_2)\pi_1$

2. The open loop Nash equilibrium

When both authorities enter into binding commitments, the open loop Nash equilibrium strategies derive from simultaneous solution of two optimal control problems. The appropriate Hamiltonan for each authority is :

(A22) $H^{M}_{t} = \alpha^{t}/2(m^{2}_{t}+\sigma\pi^{2}_{t}+\tau d^{2}_{t})$

$+ \alpha^{t+1}[p^{M}_{t+1}(rd_t+f_t-m_t-d_{t+1}) + q^{M}_{t+1}(a\pi_t+bf_t+cm_t-\pi_{t+1})]$

(A23) $H^{F}_{t} = \beta^{t}/2(f^{2}_{t} + \phi\pi^{2}_{t}+\Gamma d^{2}_{t})$

$+ \beta^{t+1}[p^{F}_{t+1}(rd_t+f_t-m_t-d_{t+1}) + q^{F}_{t+1}(a\pi_t+bf_t+cm_t-\pi_{t+1})]$

where p^{M}_{t} (resp p^{F}_{t}) et q^{M}_{t} (resp q^{F}_{t}) are the costate variables which the central bank (resp. the fiscal authority) associates with public debt and inflation.

Hence the FOCs for a minimum in period t may be written :

(A24) $\delta H^{M}_{t}/\delta m_t = m_t - \alpha(p^{M}_{t+1} - cq^{M}_{t+1}) = 0$

(A25) $\delta H^F_t/\delta f_t = f_t + \beta(p^F_{t+1} + bq^F_{t+1}) = 0$

(A26) $\delta H^M_t/\delta d_t = \tau d_t + \alpha r p^M_{t+1} = p^M_t$

(A27) $\delta H^F_t/\delta d_t = \Gamma d_t + \beta r p^F_{t+1} = p^F_t$

(A28) $\delta H^M_t/\delta \pi_t = \sigma \pi_t + \alpha a q^M_{t+1} = q^M_t$

(A29) $\delta H^F_t/\delta \pi_t = \emptyset \pi_t + \beta a q^F_{t+1} = q^F_t$

with initial and terminal conditions :

(A30a) d_1, π_1 are given

(A30b) $p^M_3 = \tau d_3$

(A30c) $p^F_3 = \Gamma d_3$

(A30d) $q^M_3 = \sigma \pi_3$

(A30e) $q^F_3 = \emptyset \pi_3$

2.1. The solution in period 2

In period 2, it is easy checked that the FOCs lead to the same strategies as above :

(A31) $m^o_2 = G_o d_2 + G_2 \pi_2$

(A32) $f^o_2 = H_o d_2 + H_2 \pi_2$

Hence :

(A33) $d^o_3 = (r+H_o-G_o)d_2 + (H_2-G_2)\pi_2$

(A34) $\pi^o_3 = (bH_o + cG_o)d_2 + (a+bH_2+cG_2)\pi_2$

Knowing m_2 and f_2, we determine the costate variables :

(A35) $p^M_2 = U_o d_2 + U_2\pi_2$

(A36) $p^F_2 = W_o d_2 + W_2\pi_2$

(A37) $q^M_2 = V_o d_2 + V_2\pi_2$

(A38) $q^F_2 = z_o d_2 + z_2\pi_2$

with

$U_o = \tau(1+\alpha r(r+H_o-G_o))$

$W_o = \Gamma(1+\beta r(r+H_o-G_o))$

$V_o = \sigma(\alpha a(bH_o+cG_o))$

$Z_o = \phi(\beta a(bH_o+cG_o))$

$U_2 = \tau(\alpha r(H_2-G_2))$

$W_2 = \Gamma(\beta r(H_2-G_2))$

$V_2 = \sigma(1+\alpha a(a+bH_2+cG_2))$

$Z_2 = \phi(1+\beta a(a+bH_2+cG_2))$

2.2. The solution in period 1

Using the FOCs (A24) and (A25), we get m_1 and f_1 in terms of d_1 and π_1 :

(A39) $m_1 = (K_o d_1+K_1 f_1+K_2\pi_1)/K_3$

(A40) $f_1 = (L_o d_1+L_1 m_1+L_2\pi_1)/L_3$

with $K_o = \alpha r(U_o-cV_o), K_1 = \alpha(U_o+bU_2-c(V_o+bV_2)),$
$K_2 = \alpha a(U_2-cV_2), K_3 = 1+\alpha(U_o-cV_o-c(U_2-cV_2)),$
$L_o = -\beta r(W_o+bZ_o), L_1 = \beta(W_o-cW_2+b(Z_o-cZ_2)),$
$L_2 = -\beta a(W_2+bZ_2), L_3 = 1+\beta(W_o+bZ_o+b(W_2+bZ_2))$

We finally get :

(A41) $m^o_1 = I_o d_1 + I_2 \pi_1$

(A42) $f^o_1 = J_o d_1 + J_2 \pi_1$

and

(A43) $d^o_2 = (r + I_o - J_o) d_1 + (I_2 - J_2) \pi_1$

(A44) $\pi^o_2 = (bI_o + cJ_o) d_1 + (a + bI_o + cJ_o) \pi_1$

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